Effect of Electric Charge on Collisions between Cloud Droplets

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ABSTRACT

A simple model is presented for the calculation of the effect of electric charges on the collision and coalescence of cloud droplets, a topic that is of importance for coalescence-induced natural rainfall and for the possible effectiveness of techniques for increasing rainfall by injection of ions into the atmosphere. Whereas electric charges of opposite sign enhance the collision efficiency of cloud droplets, the effect when all droplets bear charges of the same sign depends strongly upon the droplet sizes and separations and upon the ratio of charges on each of a droplet pair. The conditions under which coalescence is increased exist for only a very small fraction of actual cloud structures.

1. Introduction

The aim of this short paper is to set out the effect of charges on the interaction between two cloud droplets, since this has major implications for the assumed effect of ionization generators on precipitation. As background, it should be noted that the median diameter of typical cloud droplets ranges from about 15 μm for continental cumulus clouds to 30 μm for maritime cumulus clouds (Fletcher 1966, chapters 6 and 7). Since a small raindrop has a diameter of at least 1 mm, this means that from tens to hundreds of thousands of droplets must somehow coalesce to form a single raindrop. The mechanics of collision and coalescence is therefore of great importance, and electrical charges could play a significant role in the initial stages, although subsequent collisions are largely governed by aerodynamic and viscous forces as the droplets fall through the surrounding air. There has been extensive examination of these aerodynamic influences published in the literature, as documented by the present author and others (Mason 1957; Fletcher 1966, chapters 6 and 7; Pruppacher and Klett 1978, 14–17) many years ago, and no doubt in many more recent studies. Our present concern, however, is simply with the added influence of electrical charges carried on the droplets. This is motivated by the pioneering work of B. Vonnegut and his colleagues (Vonnegut and Moore 1959; Vonnegut et al. 1961, 1962a,b), who introduced large concentrations of ions into the atmosphere with the aim of modifying cloud droplet behavior.

The main content of the present paper was also investigated in a paper by Paluch in 1970 (Paluch 1970), but these calculations were much more complex since aerodynamic effects were included and not just the collision-enhancing effect of the droplet charges. For this reason we make no comparison with the results of this earlier complex calculation. Much more recently, Khain et al. (2004) have proposed techniques by which rainfall can be enhanced by the injection of charged droplets into the clouds. In retrospect, the treatment developed by Khain et al. really covers most of what is set out in the present paper but, since it is embedded in a much more complex background, the charge effects are rather less prominent. What is presented here is, in fact, the simplest possible approach to the problem and leads to readily understandable and applicable results, at least to first-order accuracy.

2. A simple theory

For a simple model relating to the collision of two droplets, it is helpful to calculate the electrical force between them if one or both are carrying an electric charge. Suppose the droplets have radii r_1 and r_2 and that they are carrying charges Q_1 and Q_2. The force between them can be calculated to a reasonable approximation in a sequence of steps, as follows.

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a. Monopole–monopole forces

Denote the distance between the centers of the two droplets by R; then, the field of droplet 1 at the center of droplet 2 is $E_1 = Q_1/(4\pi\epsilon_0 R^2)$, where $\epsilon_0 \approx 8.8 \times 10^{-12}$ F m^{-1} is the permittivity of free space. The first-order force between the two droplets is then

$$F_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}. \qquad (1)$$

This force is repulsive if positive and attractive if negative, a convention that will be preserved throughout the following sections.

b. Monopole–dipole forces

We now take note of the fact that the water constituting the droplets has a large dielectric constant ϵ, actually about 80, so that the electric field tends to polarize the droplets and create induced dipoles. The dipole produced in droplet 2 has an approximate magnitude of

$$\mu_2 \approx r_2^3 Q_1 [1 - (1/\epsilon)]/4R^2 \approx Q_1 r_2^3/4R^2. \qquad (2)$$

The sign of this dipole is such that it is attracted toward droplet 1 with force $\mu_2 dE_1/dR \approx \mu_2 Q_1/4\pi\epsilon_0 R^3$, E_1 being the electric field of droplet 1 at the position of droplet 2. There is a similar monopole–dipole force generated by polarization of droplet 1, so that the total repulsive force from this monopole–dipole interaction is

$$F_2 \approx -\frac{|\mu_2 Q_1| + |\mu_1 Q_2|}{2\pi\epsilon_0 R^3} \approx -\frac{Q_1^2 r_2^3 + Q_2^2 r_1^3}{8\pi\epsilon_0 R^5}. \qquad (3)$$

Note that this force is independent of the signs of the charges and always attractive because each dipole is induced by the other monopole.

c. Dipole–dipole forces

At the next level of approximation the forces between the induced dipoles on the droplets are taken into account. Because of their method of mutual creation, the dipoles are aligned and their polarities are such that if Q_1 and Q_2 have the same sign, then the orientation of the two dipoles will be opposite and they will repel each other, the force being $F_3 \approx \mu_1\mu_2/(4\pi\epsilon_0 R^4)$ or

$$F_3 \approx \frac{Q_1 Q_2 r_1^3 r_2^3}{64\pi\epsilon_0 R^8}. \qquad (4)$$

If the charges are opposite in sign, then the force will be attractive.

d. Combined force

It is clear that this process of calculation could be extended to higher levels, the next step being to involve quadrupoles. However, quite apart for the approximations used here, there is another major simplification, which is to assume that the droplets remain spherical. In fact they will be distorted, but this is a complex matter that does not need discussion here. It is first appropriate to add the contributions from the mechanisms so far introduced. These forces can be combined to give

$$F \approx \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} - \frac{Q_1^2 r_2^3 + Q_2^2 r_1^3}{8\pi\epsilon_0 R^5} + \frac{Q_1 Q_2 r_1^3 r_2^3}{64\pi\epsilon_0 R^8}. \qquad (5)$$

From the general form of Eq. (5), several general conclusions can be drawn. While the second R^{-5} term is always negative, implying an attractive force between the droplets, the first term, which is dominant in magnitude for larger R values, is attractive only if the charges on the two droplets are opposite. The third term, which is relevant only at very small separations, is also attractive only if the charges have opposite signs. Equation (5) is only an approximation, since there is actually an infinite sequence of terms with higher powers of r/R, but these are not really relevant since the equation ceases to be applicable once $(r_1 + r_2) > R$ and the droplets begin to overlap.

The case in which the charges Q_1 and Q_2 are of opposite sign requires no further consideration here, since all three terms in Eq. (5) are negative and the droplets are attracted to each other at all distances. Our discussion is therefore limited to the case in which the charges are of the same sign and the question of interest is the separation below which the force becomes attractive.

In the simplest case we assume that the droplets are of the same size, so that $r_1 = r_2 = r$, but carry different charges Q_1 and Q_2. Setting $F = 0$ in Eq. (5) and multiplying by $4\pi\epsilon_0 R^8/r^6$ gives a quadratic equation for which the solution is

$$\frac{R}{r} = \left[\frac{B \pm (B^2 - 4AC)^{1/2}}{2A}\right]^{1/3}, \qquad (6)$$

where $A = Q_1 Q_2$, $B = (Q_1^2 + Q_2^2)/2$, and $C = Q_1 Q_2/16$. This attractive force has a practical implication only if R/r as determined by Eq. (6) is greater than 2, since $2r$ is the separation between the droplets when they just touch. If we set $Q_2/Q_1 \equiv x$ and choose the positive sign in Eq. (6), the negative sign being omitted since it only gives R/r values less than 2, then after a little algebra the solution becomes

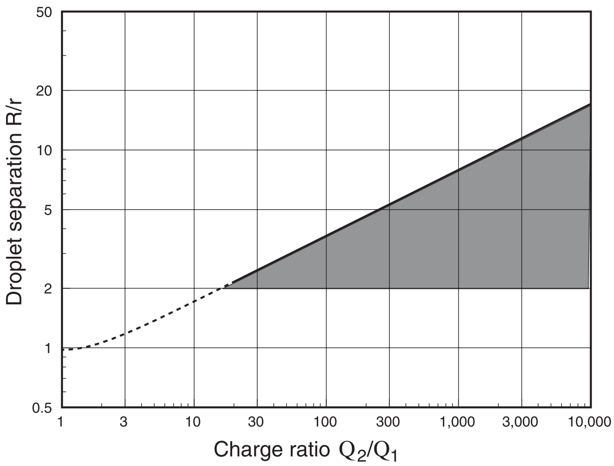

FIG. 1. Effect of the charge ratio Q_2/Q_1 on the sign of the force between two equal-sized droplets, this force being attractive within the shaded region. The range $1 < R/r < 2$ is excluded because it implies overlap between the droplets. The whole figure is reflected to the left of the vertical axis for $Q_2/Q_1 < 1$.

$$\frac{R}{r} = \left[\frac{1 + x^2 + (1 + x^2 + x^4)^{1/2}}{4x}\right]^{1/3}. \tag{7}$$

This solution in Eq. (7) remains unaltered if we interchange Q_1 and Q_2, or equivalently replace x by $1/x$. This relation is plotted for the case $Q_2 > Q_1$ in Fig. 1. Solutions with $R/r < 2$ must be ruled out since the droplets then overlap, which requires that Q_2/Q_1 be greater than about 15, but the force between droplets is attractive throughout the whole shaded region of the plot. From Fig. 1, or from Eq. (7) for large x, it can be seen that a good approximation to the separation line at the upper edge of the shaded region is that

$$\frac{R}{r} \approx 0.8\left(\frac{Q_2}{Q_1}\right)^{1/3}, \tag{8}$$

while the lower edge is at $R/r = 2$. The whole plot can be reflected horizontally about the axis $Q_2/Q_1 = 1$ to show identical behavior for $Q_2/Q_1 < 1$, as is to be expected.

Another simple case of interest is that in which one of the droplets is charged and the other uncharged so that $Q_1 = 0$. This leads to the conclusion that the first and third terms in Eq. (5) become zero and there remains only one of the numerator terms in the second term. From the sign of this term and the fact that it is proportional to Q_2^2, it always results in an attractive force, which decreases about as $1/R^5$, this force being the interaction between the charged droplet and the polarization dipole that it generates on the other droplet, as described in Eq. (3). This essentially corresponds to the limit $Q_2/Q_1 \to \infty$ in Fig. 1.

3. Droplet relative size

After the initial set of droplet collisions, the two droplets of practical interest will no longer be of the same size or carry the same charge, generally the larger droplet carrying a higher charge because it has grown through coalescence with small charged droplets. The force between a large droplet of radius r_1 and charge Q_1 and a smaller droplet of radius r_2 and charge Q_2 is given by Eq. (5), at least to the degree of approximation used here. Once more, these parameters are limited by the condition $(r_1 + r_2) < R$, where R is the distance between the droplets. In order for the force to be attractive, Eq. (5) requires that

$$F \approx \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} - \frac{Q_1^2 r_2^3 + Q_2^2 r_1^3}{8\pi\epsilon_0 R^5} + \frac{Q_1 Q_2 r_1^3 r_2^3}{64\pi\epsilon_0 R^8} < 0. \tag{9}$$

For a simple limiting case we may suppose that $r_1 \gg r_2$ so that $r_2/R \approx 0$, and Eq. (9) then leads to the approximate requirement

$$\frac{r_1}{R} > \left(\frac{2Q_1}{Q_2}\right)^{1/3}. \tag{10}$$

If the two charges Q_1 and Q_2 are of opposite sign, then Eq. (10) is satisfied for all values of the ratio r_1/R and the small droplet is always attracted to the large one, as expected. If the charges are of the same sign, then, since necessarily $r_1 < R$, the small droplet can be attracted toward the large droplet only if its charge Q_2 is greater than $2Q_1$, which is 2 times the charge on the large droplet. This is because the only significant attractive component in the force between the droplets comes from the interaction between the small droplet charge and the dipole that it induces on the large droplet.

This conclusion is very significant within the context of charged droplet coalescence, since one would expect a larger droplet that has been formed by coalescence of smaller charged droplets to carry a charge that is of the same sign but significantly larger than that on the individual droplets. This should then cause the large droplet to repel the smaller ones and so limit the coalescence process.

4. Conclusions

This simple analysis leads to the conclusion that, in order for electrical charging to enhance the collision and coalescence of cloud droplets, either the droplets must carry charges of opposite sign, which is very unlikely generally and particularly for droplets artificially charged by a corona-generating apparatus, or some of the droplets must carry charges that are much greater than those of other droplets with which they might coalesce. Perhaps unfortunately, these coalescences will tend to equalize the charges on all droplets, since droplets with small charge are attracted preferentially by droplets with large charge.

The simple analysis presented here also predicts that, for droplets of unequal size bearing charges of the same sign, coalescence is favored between small droplets that are highly charged and large droplets that are only weakly charged. Such coalescences would, of course, increase the charge on the larger droplets and so inhibit further collisions.

When these conclusions are applied to the study of ionized coalescence processes for enhancing rainfall, it is important to note that, while cloud droplets passing close to the ion generator involved will probably all be charged to a similar extent, thus producing a downwind plume of equally ionized droplets, this plume will be dispersed and mixed with the surrounding air and cloud droplets by turbulent and diffusive processes. This will then lead to a situation in which there is a heterogeneous mixture of charged and uncharged droplets of varying radius but all of the same sign.

Applying the conclusions drawn from the theoretical analysis above, the nature of this mixture, particularly the distribution of droplets of varying radius and charge, is of prime importance in determining the influence that injected electric charge may have on droplet collisions and, thus, on the subsequent evolution of the cloud. One might also speculate that a more effective charge-inducing apparatus might have its polarity changed on a periodic basis with period based upon wind speed. As turbulent convection mixes the air, enhanced coalescence between droplets of opposite charge would result. This would reduce the charge on the largest droplets, allowing them to attract small droplets charged with the same sign, as well as oppositely charged droplets. The practical effectiveness of such ionization processes in increasing rainfall, however, remains uncertain at present.

REFERENCES

Fletcher, N. H., 1966: *The Physics of Rainclouds.* Cambridge University Press, 386 pp.

Khain, A., V. Arkhipov, M. Pinsky, Y. Feldman and Ya. Ryabov, 2004: Rain enhancement and fog elimination by seeding with charged droplets. Part I: Theory and numerical simulations. *J. Appl. Meteor.,* **43,** 1513–1529.

Mason, B. J., 1957: *The Physics of Clouds.* Oxford University Press, 481 pp.

Paluch, I. R., 1970: Theoretical collision efficiencies of charged cloud droplets. *J. Geophys. Res.,* **75,** 1633–1640.

Pruppacher, H. R., and J. D. Klett, 1978: *Microphysics of Clouds and Precipitation.* D. Reidel, 714 pp.

Vonnegut, B., and C. B. Moore, 1959: Preliminary attempts to influence convective electrification in cumulus clouds by introduction of space charge into the lower atmosphere. *Recent Advances in Atmospheric Electricity,* L. G. Smith, Ed., Pergamon Press, 316–322.

——, K. Maynard, W. G. Sykes, and C. B. Moore, 1961: Technique for introducing low density space charge into the atmosphere. *J. Geophys. Res.,* **66,** 823–830.

——, C. B. Moore, R. G. Semonin, J. W. Bullock, D. W. Staggs, and W. E. Bradley, 1962a: Effect of atmospheric space charge on initial electrification of cumulus clouds. *J. Geophys. Res.,* **67,** 3909–3922.

——, ——, O. E. Stout, D. W. Staggs, J. W. Bullock, and W. E. Bradley, 1962b: Artificial modification of atmospheric space charge. *J. Geophys. Res.,* **67,** 1073–1083.